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# Relating Cohesive Zone Models to Linear Elastic Fracture Mechanics

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# RELATING COHESIVE ZONE MODELS TO LINEAR ELASTIC FRACTURE MECHANICS

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#### Abstract

The conditions required for a cohesive zone model (CZM) to predict a failure load of a cracked structure similar to that obtained by a linear elastic fracture mechanics (LEFM) analysis are investigated in this paper. This study clarifies why many different phenomenological cohesive laws can produce similar fracture predictions. Analytical results for five cohesive zone models are obtained, using five different cohesive laws that have the same cohesive work rate (CWR-area under the traction-separation curve) but different maximum tractions. The effect of the maximum traction on the predicted cohesive zone length and the remote applied load at fracture is presented. Similar to the small scale yielding condition for an LEFM analysis to be valid, the cohesive zone length also needs to be much smaller than the crack length. This is a necessary condition for a CZM to obtain a fracture prediction equivalent to an LEFM result.

#### Introduction

The origin of the cohesive zone models (CZMs) can be traced back to the "Dugdale-Barrenblatt model" [1, 2]. The cohesive zone is considered to be a fracture processing zone ahead of the crack tip. For most cohesive laws, the traction-separation curves used to model the material within the cohesive zone are phenomenological, and hence, are not directly related to the physical process in the damage zone which typically is difficult to determine experimentally. Regardless, the CZM approach has been widely accepted as a computationally convenient fracture analysis tool. If the CZM approach is used in a finite element analysis, the crack initiation, growth, and direction of growth can be automatically determined. Many different cohesive laws with variances in maximum traction, maximum separation, and shape have been proposed; such as the linear softening cohesive law by Camacho and Ortiz [3], the exponential cohesive law by Needleman [4,5] and Xu and Needleman [6], the trapezoidal cohesive law by Tvergaardand and Huchinson [7], and the polynomial cohesive law by Tvergaard[8]. Researchers found these cohesive laws generally produce results that correlated well with experimental data such as the failure load and crack growth for cracked structures. For linear elastic materials, these predictions are comparable to linear elastic fracture mechanics (LEFM) results; however, the conditions under which the cohesive zone modeling approach and the

fracture mechanics approach are equivalent have not been systematically investigated. Here, the equivalence of the two modeling approaches means that they can predict the same failure load for a cracked structure.

The objective of this paper is to investigate under what conditions the cohesive zone model and the LEFM approach are equivalent. Linear softening cohesive laws are used in this study. The relationship between the CWR (area below the traction-separation curve of the cohesive law) and the J-integral value [9] is discussed. Integral equations, for using CZMs to analyze the fracture of an infinite plate with a crack under a remote tensile load, are presented [10]. An iterative numerical procedure is implemented as a MATLAB® M-file [11] for solving these equations to obtain the length of the cohesive zone ahead of the original crack, the opening displacements along the cohesive zone, and the remote applied stress at the moment of crack growth initiation. The effect of varying the maximum traction of a cohesive law on the predicted cohesive zone length and remote applied stress is investigated. The remote applied stress is used in an LEFM formula to determine the energy release rate. By comparing the energy release rate with the CWR, it is possible to determine whether the cohesive zone modeling approach is equivalent to the LEFM approach, i.e. whether it is able to produce a similar failure load for a cracked structure to that produced by the LEFM approach.

#### CWR and J-integral value

A CZM is used to study the fracture of an infinite plate with a crack length of 2a subjected to a remote applied tensile stress  $\sigma_{\infty}$  as shown in Figure 1a. Note that all the equations and analysis results presented in this paper are related to Mode I fracture of linear elastic material. In the cohesive zones (narrow bands) ahead of the crack tips, the prospective fracture surfaces are assumed to be restrained by a cohesive stress that Dugdale took to be the yield stress of the material [2]. The traction (cohesive stress)  $\sigma$  and the separation,  $\delta = u_2^+ - u_2^-$ , of the upper and lower prospective crack surfaces, where  $u_2^+$  and  $u_2^-$  are the opening displacements in the y-direction for the upper and lower surfaces, respectively, are shown in Figure 1b.

A CZM model needs a cohesive law to relate the traction to the separation at the same location along the x-axis for describing the constitutive behavior of the cohesive zone. The cohesive law used in this paper is a linear softening cohesive law as shown in Figure 1c,

$$\sigma = \sigma_c(\delta_c - \delta) / \delta_c \quad 0 \le \delta \le \delta_c$$

$$= 0 \qquad \delta > \delta_c$$
(1)

where  $\sigma_c$  is the maximum traction and  $\delta_c$  is the maximum separation. The cohesive work rate (CWR)  $\Phi_c$ , or work of separation per unit area, is the area under the linear softening curve

$$\Phi_c = \frac{1}{2}\sigma_c\delta_c \tag{2}$$

At the moment of the crack growth initiation, the cohesive zone opening displacement at the crack tip reaches maximum separation,  $\delta(a) = \delta_c$ , and the cohesive zone is fully developed as shown in Figure 2 [7]. During the subsequent crack growth, the length of the cohesive zone and the opening displacement along the cohesive zone length are unchanged. The J-integral [9], taken along the boundary of a fully developed cohesive zone, the contour  $\Gamma$ , can be expressed as,

$$J = \int_{\Gamma} (W n_1 - T_i \frac{\partial u_i}{\partial x}) ds \tag{3}$$

where W is the strain energy per unit volume,  $n_1$  is directional cosine between the positive side (outward) of the normal N and the x-axis,  $T_i$  is the  $i^{th}$  component of the traction perpendicular to  $\Gamma$  in the outward direction,  $u_i$  is the  $i^{th}$  component of the displacement, ds is an arc element of  $\Gamma$ .

For contour  $\Gamma$  shown in Figure 3,  $n_1 = T_1 = 0$  and the J -integral value for a fully developed cohesive zone becomes [12]

$$J_{c} = -\int_{0}^{\rho} \sigma \left[ \frac{\partial u_{2}^{+}}{\partial x} - \frac{\partial u_{2}^{-}}{\partial x} \right] dx + J_{czt} = -\int_{0}^{\rho} \sigma \frac{\partial \delta}{\partial x} dx + J_{czt}$$

$$= \int_{0}^{\delta_{c}} \sigma d\delta + J_{czt} = \Phi_{c} + J_{czt}$$
(4)

where  $J_c$  is the critical J value immediately before the initiation of crack growth and  $J_{czt}$  is a J-integral value of an infinitesimal contour, not shown in the figure, around the cohesive zone tip. For an isotropic material,  $J_{czt}$  is related to the stress intensity factor  $K_{czt}$ , if it exists, at the cohesive zone tip

$$J_{czt} = \frac{K_{czt}^{2}}{E} \quad . \tag{5}$$

Since the J-integral is a path independent integral, the J value obtained from any contour integral around the cohesive zone such as  $\Gamma_1$  in Figure 3 must also equal to  $J_c$ . If the cohesive zone model is equivalent to an LEFM model, the stress field outside of the cohesive zone must

be near the same as the K-dominant stress field [13] in the LEFM model. In other words, the path independent  $J_c$  can be related to the fracture toughness  $K_c$  as

$$K_c = \sqrt{EJ_c} \tag{6}$$

and the critical energy release rate defined in LEFM as

$$G_c = J_c \quad . \tag{7}$$

For cohesive laws that assume the CWR to be the same as the critical energy release rate,

$$\Phi_c = G_c = J_c \tag{8}$$

the stress intensity factor at the cohesive zone tip,  $K_{czt}$  in Eq. (5), must vanish because  $J_{czt}$  must be zero to satisfy Eq. (8). Note that  $G_c$  and  $K_c$  are material properties and are experimentally determined, so  $\Phi_c$  is also considered to be a material property.

Because  $J_{czt} = 0$  in Eq. (4), then  $J_c$  obtained by the contour around the cohesive zone is the CWR  $\Phi_c$  corresponding to the area under the traction-separation curve. Cohesive zone models with the same CWR but different shapes will produce the same  $J_c$ . If  $J_c$  is the only parameter to determine fracture, it is expected that the same failure prediction can be obtained with different shapes of the cohesive law. This may be the reason why CZMs with different shapes can generate similar failure predictions.

#### **Analytical Solutions for Cohesive Zone Models**

CZM analyses of a cracked infinite plate subjected to a remote tensile stress, as shown in Figure 1a, are presented in this section. Five linear softening cohesive laws shown in Figure 4, with different maximum tractions but all having the same CWR, were used in this study. The effects of maximum traction on the predicted cohesive zone length and remote applied stress were investigated. The predicted remote applied stresses were used in an LEFM formula to compute the energy release rate at failure. Good agreement between the energy release rate at failure and the CWR determines the equivalence of a CZM and a LEFM analysis.

The analytical procedure presented here follows the paper authored by Jin and Sun [10]. The cohesive zone is treated as an extended part of the crack with the stress intensity factor being removed at the tip of the cohesive zone. Hence,

$$\sigma_{\infty}\sqrt{\pi c} - \frac{2}{\sqrt{\pi c}} \int_{a}^{c} \frac{\sigma(\xi)d\xi}{\sqrt{1 - \xi^2/c^2}} = 0 \tag{9}$$

where c is the x-coordinate of the cohesive zone tip and  $\sigma(\xi)$  is the traction at location  $\xi$  as shown in Figure 1b.

The total cohesive zone opening displacement  $\delta$  at location x is [14]

$$\delta(x) = \frac{4c\sigma_{\infty}}{E} \sqrt{1 - \frac{x^2}{c^2}} - \frac{4}{\pi E} \int_a^c G(x, \xi) \sigma(\xi) d\xi$$
 (10)

where E is the Young's modulus, and  $G(x,\xi)$  is an influence function given by

$$G(x,\xi) = \ln \left| \frac{\sqrt{1 - x^2 / c^2} + \sqrt{1 - \xi^2 / c^2}}{\sqrt{1 - x^2 / c^2} - \sqrt{1 - \xi^2 / c^2}} \right|$$
(11)

For the linear softening model shown in Figure 1b, Eq. (10) can be expressed as

$$\delta(x) = \frac{4c\sigma_{\infty}}{E} \sqrt{1 - \frac{x^2}{c^2}} - \frac{4}{\pi E} \int_a^c G(x, \xi) \frac{\sigma_c(\delta_c - \delta(\xi))}{\delta_c} d\xi$$
 (12)

and Eq. (9) can be expressed as

$$\sigma_{\infty} = \frac{2\sigma_{c}}{\pi c} \left( -\int_{a}^{c} \frac{\delta(\xi)/\delta_{c}}{\sqrt{1 - \xi^{2}/c^{2}}} d\xi + \int_{a}^{c} \frac{d\xi}{\sqrt{1 - \xi^{2}/c^{2}}} \right)$$
(13)

Substituting  $\sigma_{\infty}$  into Eq. (12), an integral equation is obtained for determining the cohesive zone length,  $\rho = c - a$ , and cohesive zone opening displacements (CZODs)  $\delta(x)$ 

$$\frac{\pi E \delta^*(x) \Phi_c}{2c\sigma_c^2} - \frac{1}{c} \int_a^c G(x,\xi) \delta^*(\xi) d\xi + \frac{2}{c} \sqrt{1 - \frac{x^2}{c^2}} \int_a^c \frac{\delta^*(\xi)}{\sqrt{1 - \xi^2/c^2}} d\xi$$

$$= \frac{2}{c} \sqrt{1 - \frac{x^2}{c^2}} \int_a^c \frac{d\xi}{\sqrt{1 - \xi^2/c^2}} - \frac{1}{c} \int_a^c G(x,\xi) d\xi$$
(14)

where  $\delta^*(x) = \delta(x) / \delta_c$ .

Equation 14 can be solved with an iterative method shown in Figure 5, in which an initial cohesive zone length  $\rho = c - a$  is given first to determine the integral interval, [a,c], for the

integral terms in Eq. (14) and then the CZODs along c-a are computed. If the predicted opening displacement at the crack tip  $\delta(a)$  is not equal to the maximum separation  $\delta_c$ , an updated cohesive length is given until the  $\delta(a)$  equals  $\delta_c$ . The final value of c-a is the cohesive zone length  $\rho$ . The remote applied stress  $\sigma_\infty$  can be obtained with Eq. (13), by using the final set of CZODs. The remote applied stress is used to compute the energy release rate using the following LEFM formulae,

$$G(\sigma_{\infty}) = \frac{\pi a \sigma_{\infty}^{2}}{F} \tag{15}$$

If the ratio of the computed energy release rate  $G(\sigma_{\infty})$  to the CWR,  $\Phi_c$ , is close to unity, then the CZM is considered to be equivalent to an LEFM analysis.

#### Results

Analytical results for five cohesive law models with different maximum tractions as shown in Figure 4 are presented in this section. All the results are computed at the initiation of crack growth when the cohesive zone is fully developed. The effect of maximum tractions on cohesive zone length, remote applied stress, the equivalence of CZM approach and LEFM analysis is presented. The effect of maximum traction on cohesive zone length can be found in Figure 6, which shows that larger maximum tractions result in shorter cohesive zone lengths,  $\rho$ . Two curves represent results obtained with two different crack lengths, one has  $a/l_{ch}=1$  and the other one has  $a/l_{ch}=10$ . Note that  $l_{ch}$  is a characteristic length, and in this paper it is defined based on Model B in Figure 4,

$$l_{ch} = \frac{E\Phi_c}{\sigma_c^2} \tag{16}$$

For the rest of the paper, the term crack length means the half-crack length a in Figure 1a. The two curves in Figure 6 are converged for maximum tractions greater than  $2\sigma_c$ , indicating that the cohesive zone length may not depend on the crack length for a cohesive law with a large maximum traction. However, the cohesive zone lengths depend on crack length for models with a maximum traction less than  $\sigma_c$ . The cohesive zone length for the short crack  $a/l_{ch}=1$  is longer than that of the long crack  $a/l_{ch}=10$ .

Cohesive zone lengths are predicted for all five cohesive law models in Figure 4. The cohesive zone length reaches a constant value for all cohesive law models as the crack length increases. For illustration purposes, only results of Models A, B, and D are plotted in Figure 7. Models A, B, and D have maximum tractions of  $1/2\sigma_c$ ,  $\sigma_c$ , and  $2\sigma_c$ , respectively. For the largest

maximum traction  $2\sigma_c$  model, the cohesive zone length reaches a constant value at a crack length around  $a=2l_{ch}$ . For the second largest maximum traction  $\sigma_c$  model, the cohesive zone length reaches a constant value at a crack length around  $a=5l_{ch}$ . The cohesive zone length almost reaches a constant value at  $a=16l_{ch}$  for the model with the smallest maximum traction of 1/2  $\sigma_c$ .

The cohesive zone opening displacements along the cohesive zone length for models with different maximum tractions are plotted in Figure 8. There are three cohesive zone opening displacement curves associated with three cohesive laws with maximum tractions of  $1/2\sigma_c$ ,  $\sigma_c$ , and  $2\sigma_c$ , respectively. These cohesive zone opening displacement curves along the cohesive zone length are not linear (the shape of the cohesive zone is a cusp). Figure 8 also shows that the smaller the maximum traction results in the longer the cohesive zone length. These opening displacements are used in Eq. (13) to compute the remote applied stresses.

The computed remote stresses as a function of the maximum traction of all five CZM models are plotted in Figure 9. There are two curves shown on Figure 9 representing two different crack lengths. The bottom curve shows that the remote applied stresses for the longer crack length of  $a = 10l_{ch}$  reaches a constant value for maximum traction greater than one  $\sigma_c$ , but the top curve for the shorter crack length of  $a = 1l_{ch}$  shows that the remote applied stresses has not reached a constant valve for the maximum traction at  $4\sigma_c$ .

The computed remote applied stresses are then used in Eq. (15) to compute LEFM energy release rates  $G(\sigma_{\infty})$ . The ratio of  $G(\sigma_{\infty})$  to  $\Phi_c$  is defined as the normalized LEFM energy release rate (NLERR). If the NLERR is near unity, then the cohesive zone model is equivalent to an LEFM analysis, and both models would predict the same failure load. The NLERR as a function of maximum traction is plotted in Figure 10. The figure shows that for a long crack length,  $a = 10l_{ch}$ , the NLERR approaches unity for maximum tractions greater than  $2\sigma_c$  while for a short crack length,  $a = l_{ch}$ , the NLERR reaches unity at a much slower rate (a cohesive law with a maximum traction greater than  $4\sigma_c$  needs be used).

The NLERR requires high tractions to reach unity for the shorter crack length  $a = l_{ch}$  because the cohesive zone length relative to the crack length is not small ( $\ll 1$ ). This large cohesive zone length, similar to a large scale yielding zone, may significantly alter the original K dominant stress field around the crack tip [13]. Hence, the results from the CZM model cannot be equivalent to an LEFM analysis. To confirm this hypothesis, the NLERR as a function of the ratio of cohesive zone length to crack length is plotted in Figure 11 for both the crack lengths. This plot reveals that the two curves shown in Figure 10 can be collapsed to become a single

curve. This indicates that the ratio of cohesive zone length to the crack length may be an important parameter for fracture predictions using CZM. The ratio of cohesive zone length to crack length,  $\rho/a$ , must be close to zero for the CZM to be equivalent to an LEFM analysis. Physically, this means that regions ahead of the crack tip that exibit mechanisms other than brittle fracture (plasticity, bridging fibers, etc) must be very small relative to the crack length for LEFM to be valid, and hence, for the CZM approach to be equivalent to LEFM.

#### Summary

A MATLAB® M-file was implemented for numerically solving the CZM integral equations with an iterative procedure. Linear softening cohesive laws with a constant CWR but different maximum tractions were used for modeling the traction-separation behavior in the cohesive zone. Results show that a reduced cohesive zone length is predicted for a cohesive model with a large maximum traction, and the ratio of cohesive zone length to the crack size needs be very small for the prediction from the CZM to be equivalent to an LEFM analysis.

The conditions required for a cohesive zone model and an LEFM analysis to be equivalent were identified from this study and other published literature. These conditions are as follows:

- 1. Stress intensity factor at a fully developed cohesive zone tip needs to vanish if the J-integral value around the cohesive zone is set equal to the cohesive work rate.
- 2. If the J-integral value around a fully developed cohesive zone is the only parameter to determine failure, then the maximum traction of a cohesive law can be changed while keeping the CWR constant without affecting the failure load prediction.
- 3. The ratio of the cohesive zone length to the crack length needs to be small, so the existence of the cohesive zone cannot significantly change the stress field near the crack tip.
- 4. The maximum traction of a CZM needs to be high. High maximum tractions result in a short cohesive zone length.

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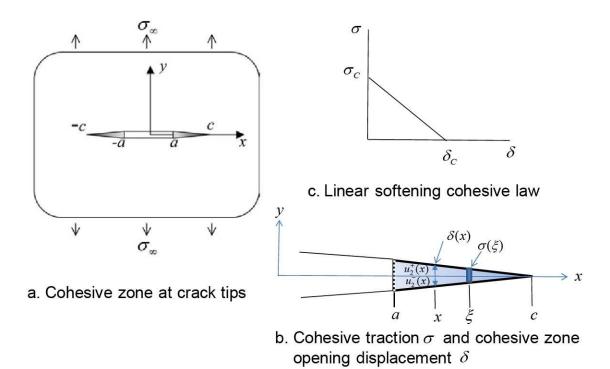


Figure 1 Fracture analysis of a cracked infinite plate using a cohesive zone model.

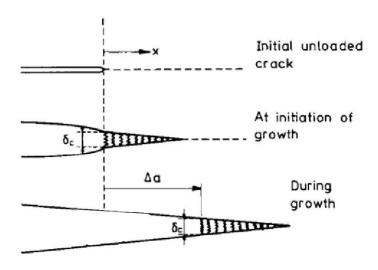


Figure 2 Cohesive zone fully developed at crack growth initiation and unchanged during growth [7].

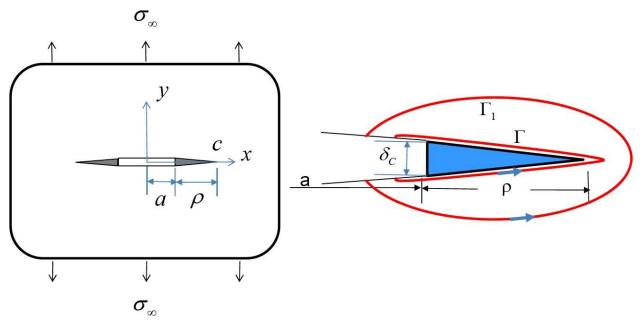


Figure 3 J-integral paths around the cohesive zone.

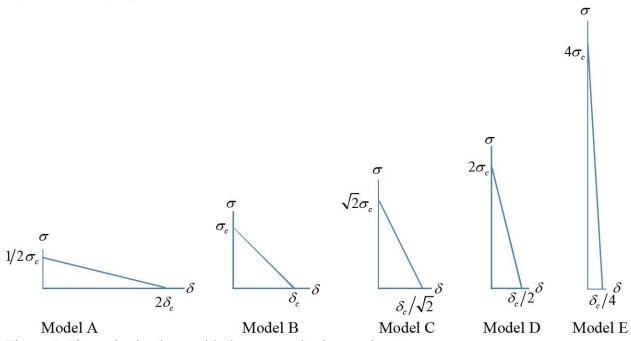


Figure 4 Five cohesive laws with the same cohesive work rate.

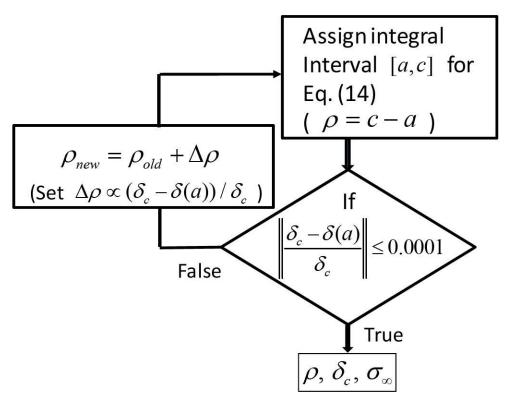


Figure 5 Solution procedure of the iterative method.

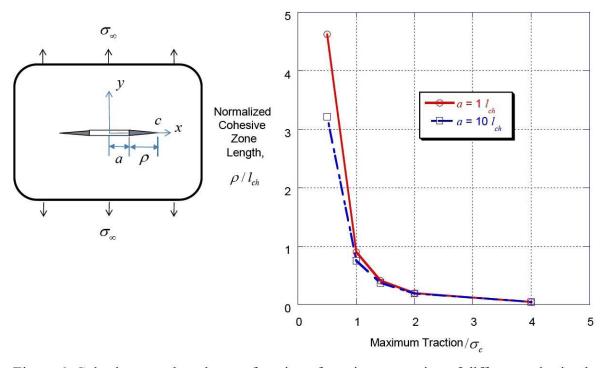


Figure 6 Cohesive zone lengths as a function of maximum traction of different cohesive laws.

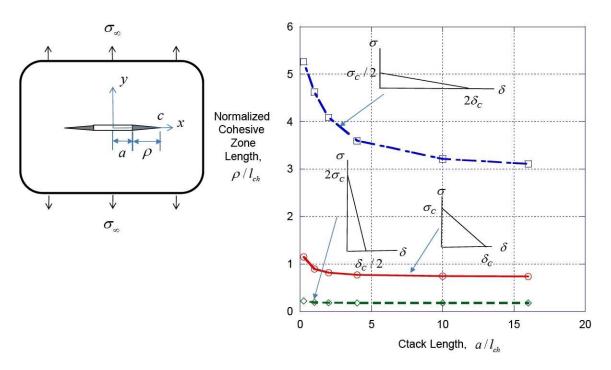


Figure 7 Cohesive zone lengths vs. crack length for models with different maximum tractions.

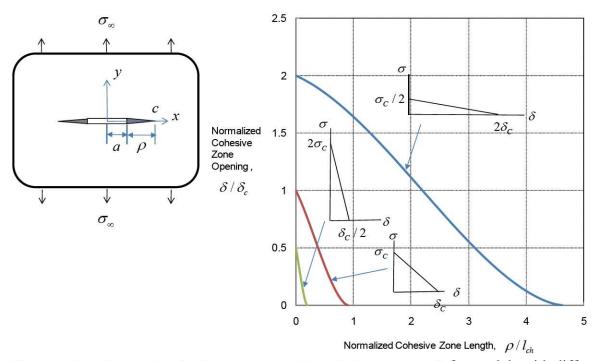


Figure 8 Cohesive opening displacements along the cohesive zone length for models with different maximum tractions ( $a = 1l_{ch}$ ).

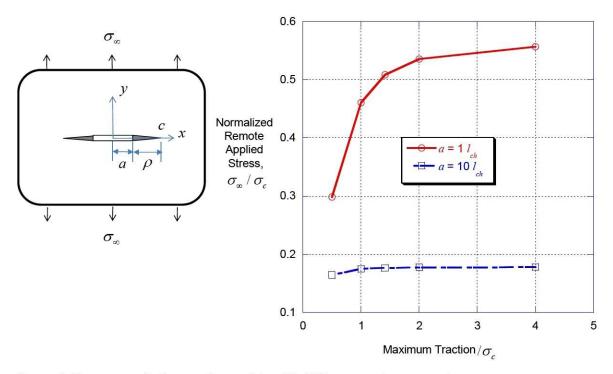


Figure 9 Remote applied stress for models with different maximum tractions.

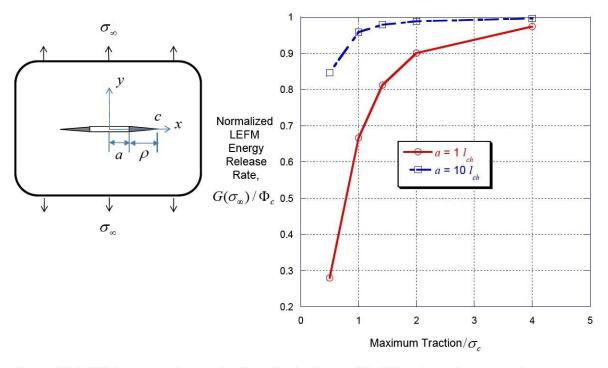


Figure 10 LEFM energy release rates for cohesive laws with different maximum tractions.

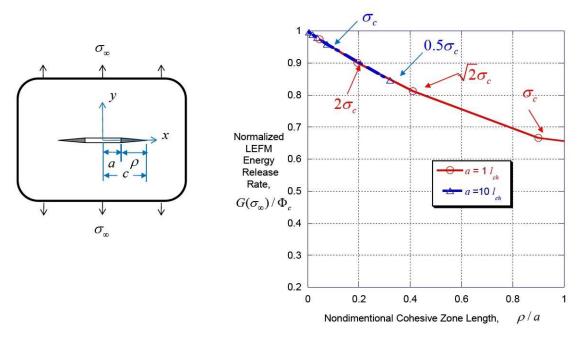


Figure 11 LEFM energy release rates at the moment of growth initiation as a function of the ratio of cohesive zone length to crack length.

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#### 13. SUPPLEMENTARY NOTES

#### 14. ABSTRACT

The conditions required for a cohesive zone model (CZM) to predict a failure load of a cracked structure similar to that obtained by a linear elastic fracture mechanics (LEFM) analysis are investigated in this paper. This study clarifies why many different phenomenological cohesive laws can produce similar fracture predictions. Analytical results for five cohesive zone models are obtained, using five different cohesive laws that have the same cohesive work rate (CWR-area under the traction-separation curve) but different maximum tractions. The effect of the maximum traction on the predicted cohesive zone length and the remote applied load at fracture is presented. Similar to the small scale yielding condition for an LEFM analysis to be valid, the cohesive zone length also needs to be much smaller than the crack length. This is a necessary condition for a CZM to obtain a fracture prediction equivalent to an LEFM result.

#### 15. SUBJECT TERMS

Cohesive zone model, Linear elastic fracture mechanics, Cohesive work rate, Energy release rate

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